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## TUNING OF IMC – PID CONTROLLER FOR OPTIMIZED CONTROL

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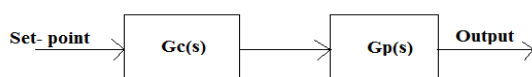
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**Abstract:** Time delays are usually unavoidable in the engineering systems like mechanical and electrical systems etc. The presence of delay causes unwanted impacts on the system under consideration which imposes strict limitations on achievable or targeted feedback performance in both continuous and discrete systems. The presence of the delay complicates the design process as well. It makes continuous systems to be infinite dimensional and also it significantly increases the dimensions in discrete systems. As the internal model control based proportional integral derivative controller are simple and robust to handle the model uncertainties and disturbances. But they are less sensitive to noise than proportional integral derivative controller for an actual process in industries. It results in only one tuning parameter which is closed loop time constant  $\lambda$  internal model controller filter factor. It also provides a good solution to the process with significant time delays which is actually the case with working in real time environment. So in this paper internal model control based proportional integral derivative controller is designed. The Pade's approximation for the time delay has been used because most of the controller design based on different methods can not be used with the delayed systems. While comparing the responses of the transfer functions of different kinds of orders the internal model control based proportional integral derivative controller will not give the same results as the internal model control strategy because of approximation used for delay time. Also the standard internal model filter from  $f(s) = 1 / (\lambda s + 1)$  shows good set point tracking. Thus internal model control based proportional integral derivative controller is able to compensate for disturbances and model uncertainty while open loop control is not. Internal model control is also detuned to assure stability even if there is model uncertainty.

**Keywords:** IMC, IMC–PID method, IMC–PID tuning with time delay

### 1. INTRODUCTION

The model based control systems are often used to track set points and reject low disturbances. The internal model control internal model control is based on the principle of the internal model. It states that if any control system contains within it implicitly or explicitly some representation of the process to be controlled then a perfect control is easily achieved. If the control scheme has been developed based on the exact model of the process then it is possible to get perfect control theoretically. The open loop control strategy is shown in figure 1.



**Figure 1:** Open loop control strategy

For above open loop control system:

Output is  $G_c \cdot G_p$ .

Set-point multiplication of all three parameters

$G_c$  is the controller of process

$G_p$  is the actual process or plant

$G_p^*$  is the model of the actual process or plant

A controller  $G_c$  is used to control the process  $G_p$ .

Suppose  $G_p^*$  is the model of  $G_p$  then by setting.

$G_c = \text{inverse of } G_p^*$  inverse of model of the actual process

And if

$G_p$  is  $G_p^*$  the model is the exact representation of the actual process

The perfect control on the process can be achieved by having the complete knowledge of the process. The ideal control performance is achieved without feedback which signifies that feedback control is necessary only when knowledge about the process is inaccurate or incomplete.

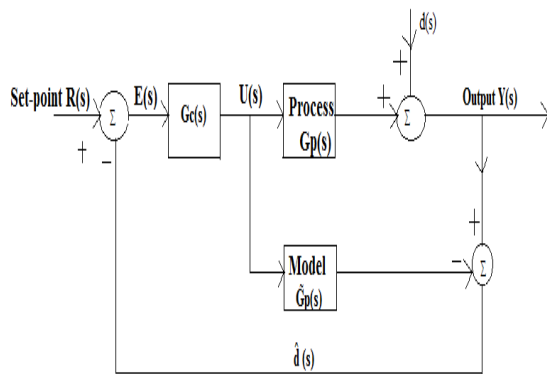
The internal model control design procedure is identical to the open loop control design procedure. The implementation of internal model control results in a feedback system. So the internal model control is able to compensate for disturbances and model uncertainty while open loop control is not. The internal model control also detuned to assure stability if there is model uncertainty [7], [3].

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## 2. INTERNAL MODEL STRATEGY

The process-model mismatch is common. The process model may not be invertible and the system is often affected by unknown disturbances. So the above open loop control arrangement will not be able to maintain output at setpoint. It forms the basis for the development of a control strategy that has the potential to achieve perfect control. This strategy is known as internal model control IMC. The general structure shown in Fig. 2



**Figure 2:** Schematic of the internal model control scheme

$d(s)$  is an unknown disturbance which affects the system as shown in the figure 2

$U(s)$  is the manipulated input which is introduced to both the process and its model

$Y(s)$  is the process output compared with the output of the model, resulting in a signal  $\hat{d}(s)$ .

$$Y(s) = [G(s) - p(s)] U(s) + d(s) \quad (1)$$

$$\hat{d}(s) = Y(s) - p(s)U(s) \quad (2)$$

Then the  $\hat{d}(s)$  is the difference in behaviour between the process and its model.

$$\hat{d}(s) = Y(s) - p(s)U(s) \quad (3)$$

$\hat{d}(s)$  is equal to the unknown disturbance.

The  $\hat{d}(s)$  is missing in the model  $p(s)$  so can be used to improve control. This is done by subtracting  $\hat{d}(s)$  from the setpoint  $R(s)$  which is very similar to affecting a setpoint trim. The resulting control signal is given by

$$E(s) = [R(s) - \hat{d}(s)] G_c(s) \quad (4)$$

$$U(s) = [R(s) - \hat{d}(s)] G_c(s) / [1 + [G(s) - p(s)] G_c(s)] \quad (5)$$

$$Y(s) = G_p(s) U(s) + d(s) \quad (6)$$

The closed loop transfer function for the internal model control scheme is given by

$$Y(s) = \frac{R(s) - \hat{d}(s)] G_c(s)}{1 + [G(s) - p(s)] G_c(s)} + d(s) \quad (7)$$

$$Y(s) = \frac{G_p(s) R(s) G_c(s) + 1 - [G_c(s) - p(s)] d(s)}{1 + [G(s) - p(s)] G_c(s)} \quad (8)$$

Perfect setpoint tracking and disturbance rejection is achieved if  $G_c(s) = p(s)-1$  and if  $G(s) = p(s)$ . If  $G(s) \neq p(s)$  perfect disturbance rejection can be realised provided  $G_c(s) = p(s)-1$ .

The effects of process model mismatch should be minimised to improve robustness. A low-pass filter  $G_f(s)$  is added to attenuate the effects of process-model mismatch. Since discrepancies between process and model behaviour occur at the high frequency end of the system's frequency response. The internal model controller is designed as the inverse of the process model in series with a low-pass filter, i.e.  $GIMC = G_c(s) G_f(s)$ . The order of the filter is so chosen that  $G_c(s) G_f(s)$  is proper to prevent excessive differential control action. The resulting closed loop is then given by

$$Y(s) = \frac{G_p(s) R(s) GIMC(s) + 1 - [GIMC(s) - p(s)] d(s)}{1 + [G(s) - p(s)] GIMC(s)} \quad (9)$$

### a. Practical Design of Internal Model Control

It is relatively easy to design an internal model controller.  $p(s)$  is model of the process. First factor  $p(s)$  into "invertible" and "non-invertible" components.

$$p(s) = +p(s) -p(s) \quad (10)$$

The non-invertible component,  $-p(s)$  contains terms which if inverted, will lead to instability and reliability problems terms containing positive zeros and time-delays. Set  $G_c(s) = +p(s)-1$  and then  $GIMC = G_c(s) G_f(s)$ , where  $G_f(s)$  is a low-pass function of appropriate order [7], [4].

### Internal Model Control Tuning of Proportional Integral Derivative controllers:

Internal model control IMC is a model-based control method. The internal model control method can also be used as a tuning method for the proportional integral derivative controller. Generally, the method is applicable for systems with constant delays but the internal model control method is also applied for varying time-delay

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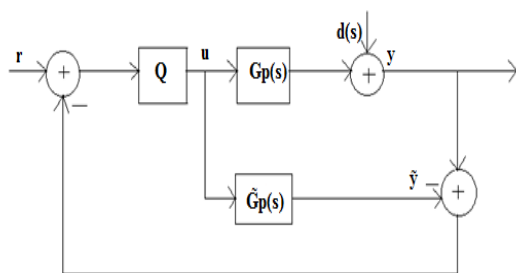
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systems. Figure 3 represents the internal model control principle. In the figure  $G_p(s)$  is the process controlled

$Q$  is the IMC controller  
 $p(s)$  is the process model  
 $d(s)$  is the disturbance  
 $y$  is the process output

$r$  is the set point

The model output error  $y - \hat{y}$  is subtracted from the reference signal and fed into the internal model control controller which calculates the control signal.



**Figure 3:** IMC modified structure [1]

An internal model control controller  $Q(s)$  is calculated so that the process model is first divided into two parts

$$p(s) = p_+(s) p_-(s) \quad (11)$$

$p_+(s)$  is the non-invertible part of the model including all unstable zeros and delays. The rest of the model is included in  $p_-(s)$ . The controller is given as

$$Q(s) = \frac{1}{p_-(s)} f(s) \quad (12)$$

where  $f$  is a low-pass filter transfer function of order  $n$  given as

$$f(s) = \frac{1}{(1 + s \lambda_{IMC})^n} \quad (13)$$

The low-pass filter is required in order to have a causal controller. The  $\lambda_{IMC}$  is the tuning parameter of the internal model control method. The value of  $\lambda_{IMC}$  has a significant effect on the performance and robustness of the controlled system. There is a trade-off a very fast and simultaneously very robust tuning is generally difficult to achieve. In varying time-delay systems where robustness with respect to delay variance plays a crucial role the tuning of  $\lambda_{IMC}$  turns out to be crucial. The dependency between the jitter margin the control system performance. So the

$\lambda_{IMC}$  parameter is further discussed where control is used in the NCS setup [5], [6].

When implementing the internal model control controller, it is useful to recognize the dependency between the internal model control controller  $Q$  in Figure 3.1 and the controller in the classical feedback loop. The internal model control law in the classical control loop is given by

$$G_c(s) = \frac{Q(s)}{1 - (s) Q(s)} \quad (14)$$

The process delays must be approximated with linear transfer functions in order to be able to calculate the controller if the controller is used. A constant delay of  $\tau$  seconds corresponds to an exponential function  $e^{-\tau s}$  in the Laplace domain, and the delay can be approximated with the Taylor series expansion or the first order Padé approximation

$$e^{-\tau s} = \frac{(1 - s \tau / 2)}{(1 + s \tau / 2)} \quad (15)$$

The internal model control design often yields high order controllers under certain assumptions. It is possible to obtain the proportional integral control structure from the internal model control design and thus get the tuning parameters for a regular proportional internal control controller. Consider the FOTD process model. Using the internal model control design and the first order Taylor series expansion  $e^{-s\tau} \approx 1 - s\tau$  with  $n = 1$  order of the low-pass filter the controller  $C$  becomes [2], [10].

$$\begin{aligned} CPI(s) &= \frac{1 + sT}{K_p s (\lambda_{IMC} + \tau)} = \frac{T}{K_p (\lambda_{IMC} + \tau)} \\ &= (1 + 1/sT) \end{aligned} \quad (16)$$

The proportional integral controller structure with parameters is given below

$$K = \frac{T}{K_p (\lambda_{IMC} + \tau)} = T_i = T \quad (17)$$

$$k_p = \frac{(1 + 1/sT)}{K_p (\lambda_{IMC} + \tau)}; \quad k_i = \frac{1}{K_p (\lambda_{IMC} + \tau)} \quad (18)$$

If the Padé approximation of the delay is used the controller  $C$  becomes



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2. Factor out the noninvertible elements to avoid the bad part all-pass

$$p^-(s) = k_p / (\tau_p s + 1) (0.5\theta s + 1) \quad (30)$$

$$p^+(s) = (-0.5\theta s + 1) \quad (31)$$

3. The idealized controller is then given by

$$Q(s) = (\tau_p s + 1) (0.5\theta s + 1) / k_p \quad (32)$$

4. The filter  $f(s)$  is added to make the  $Q(s)$  proper. But  $Q(s)$  will be make semiproper to get the proportional integral derivative controller. The derivative option will be used to allow the numerator of  $q(s)$  to be one order higher than the denominator. It is done only to obtain an ideal proportional integral derivative controller.

$$Q(s) = (s) f(s) = (1 / p^-(s)) f(s) \\ = (\tau_p s + 1) (0.5\theta s + 1) (1) / k_p (\lambda s + 1) \quad (33)$$

The proportional integral derivative equivalent can be given as

$$G_c(s) = Q(s) / 1 - p^-(s) Q(s) \\ (s) f(s) / 1 - p^-(s) f(s) Q(s) \quad (34)$$

$$(s) f(s) / 1 - p^-(s) p^+(s) (1 / p^-(s)) f(s) \\ (s) f(s) / 1 - p^+(s) f(s)$$

$$= \frac{1}{k_p} \frac{(\tau_p s + 1) (0.5\theta s + 1)}{(\lambda + 0.5\theta s) s} \quad (35)$$

$$G_c(s) = \frac{1}{k_p} \frac{0.5 \tau_p \theta s^2 + (\tau_p + 0.5\theta) s + 1}{(\lambda + 0.5\theta s) s} \quad (40)$$

Multiply equation (3.40) by  $(\tau_p + 0.5) / (\theta / \tau_p + 0.5\theta)$  PID parameters can be evaluated as shown below

$$K_c = (\tau_p + 0.5\theta) / k_p (\lambda + 0.5\theta) \quad (41)$$

$$\tau_i = \tau_p + 0.5\theta \quad (42)$$

$$\tau_D = (\tau_p \theta) / (2\tau_p + \theta) s \quad (43)$$

The internal model control based proportional integral derivative controller design procedure has resulted in a proportional integral derivative controller when the process is first-order + dead time. A Padé approximation for dead time was used in this development meaning that the filter factor ( $\lambda$ ) cannot be made arbitrarily small. So there will

be performance limitations to the internal model control based proportional integral derivative strategy that do not occur in the internal model control strategy. Rivera et.al. (1986) recommend that  $\lambda > 0.8\theta$  because of the model uncertainty due to the Padé approximation. The use of an all-pass in the factorization will lead to a proportional integral derivative controller in series with a first-order lag. The parameters in this case are shown as the first entry in Table 1. Morari and Zafiriou (1989) recommend  $\lambda > 0.25\theta$  for the PID + lag formulation. The third and fourth entries neglect the time delay in forming the PI controller [7]

Table 1: Proportional integral derivative Tuning Parameters for Stable Time-Delay Processes [7]

	$G_p(s)$	$k_c$	$\tau_i$	$\tau_D$	$\tau_f$
A	$\frac{K_p e^{-\theta s}}{\tau_p s + 1}$	$\frac{\tau_p + \theta/2}{k_p(\theta/2 + \lambda)}$	$\tau_p + \theta/2$	$\frac{\tau_p \theta}{2\tau_p + \theta}$	$\frac{\lambda \theta}{2(\lambda + \theta)}$
B	$\frac{K_p e^{-\theta s}}{\tau_p s + 1}$	$\frac{\tau_p + \theta/2}{k_p(\theta/2 + \lambda)}$	$\tau_p + \theta/2$	$\frac{\tau_p \theta}{2\tau_p + \theta}$	
C	$\frac{K_p e^{-\theta s}}{\tau_p s + 1}$	$\frac{\tau_p}{k_p \lambda}$	$\tau_p$		
D	$\frac{K_p e^{-\theta s}}{\tau_p s + 1}$	$\frac{\tau_p + \theta/2}{k_p \lambda}$	$\tau_p + \theta/2$		
E	$\frac{K e^{-\theta s}}{s}$	$\frac{2\lambda + \theta}{K(\theta + \lambda)^2}$	$2\lambda + \theta$		
F	$\frac{K e^{-\theta s}}{s}$	$\frac{2}{K(\theta/2 + \lambda)}$	$2\lambda + \theta$	$\frac{\lambda \theta + \theta^2/4}{2\lambda + \theta}$	
G	$\frac{K e^{-\theta s}}{s}$	$\frac{\theta}{K(2\lambda + \theta)}$	$\theta/2$		
H	$\frac{K e^{-\theta s}}{s}$	$\frac{\theta}{K(4\lambda + \theta)}$	$\theta/2$	$\theta/6$	$\frac{2\lambda^2 + \theta^2/6}{4\lambda + \theta}$

### Integrator + Dead Time:

For processes where the time constant is dominant, the step response behaviour can be approximated as integrator + dead time as characterized by the following transfer function.

$$G_p(s) = K e^{-\theta s} / s \quad (44)$$



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A Taylor series approximation for dead time is used. Also, the special filter form for integrating systems is used [7].

$$e^{-\theta s} = -\theta s + 1 \quad (45)$$

$$G_p(s) = k(-\theta s + 1) / s = (k / s) (-\theta s + 1) \quad (46)$$

$$Q(s) = (s / k) \cdot ((\lambda s + 1) / (\lambda s + 1)^2) \quad (47)$$

By using internal model control based proportional integral derivative procedure

$$G_c(s) = Q(s) / 1 - p(s) Q(s) \quad (48)$$

A proportional integral controller results with the following parameters

$$k_c = 2\lambda + \theta / k(\lambda + \theta)^2 \quad (49)$$

$$\tau_i = 2\lambda + \theta$$

### Gain + Dead Time:

For processes where the time delay is dominant the step response behavior can be approximated as gain + dead time as characterized by the following transfer function.

$$G_p(s) = K e^{-\theta s} \quad (50)$$

Using a second-order Padé approximation for the time delay [7]

$$e^{-\theta s} = \frac{\theta^2 s^2 / 12 - \theta s / 2 + 1}{\theta^2 s^2 / 12 + \theta s / 2 + 1} \quad (51)$$

$$G_p(s) = k \frac{\theta^2 s^2 / 12 - \theta s / 2 + 1}{(\theta^2 s^2 / 12 + \theta s / 2 + 1)}$$

$$= \frac{k}{(\theta^2 s^2 / 12 + \theta s / 2 + 1)} (\theta^2 s^2 / 12 - \theta s / 2 + 1) \quad (52)$$

The PID plus filter controller results with

$$\begin{aligned} k_c &= \theta / (4\lambda + \theta) \\ \tau_i &= \theta / 2 \\ \tau_d &= \theta / 6 \\ \tau_F &= (2\lambda^2 - \theta/6) / (4\lambda + \theta) \end{aligned} \quad (53)$$

but  $\lambda > \theta / \sqrt{2}$ .

## 3. SIMULATION AND RESULTS

### a. System Implementation:

The internal model control based proportional integral derivative controller design with time delay is implemented using matlab. The version of matlab used here is 7.13.0.564 (R20011b). The standard matlab package is useful for linear systems analysis. The version of simulink used is 7.8 (R2011b). The simulink is far more useful for control system simulation. simulink enables the rapid construction and simulation of control block diagrams.

### Ideal Internal Model Control based Proportional Integral Derivative Controller:

The actual process transfer function is never known exactly. So it is necessary to use two transfer function representations of the process. So one is considered as process or plant which is never known exactly and the second is considered as process model which is known exactly. In internal model controller process model is kept in parallel with the actual process. The ideal internal model control based proportional integral derivative controller means the model is perfect and there is no disturbance and delay. So the feedback is also nil. The equation which tells that the model is perfect is given below i.e open loop system.

$$(s) \quad (54)$$

$$d(s) = 0 \quad (55)$$

$G_p(s)$  is the process transfer function.

$(s)$  is the process model.

$d(s)$  is the disturbance.

### b. Simulation of Ideal Internal Model Control Based Proportional Integral Derivative Design:

The internal model controller provides a transparent frame work for control system design and tuning. For simulation of ideal internal model control based proportional integral derivative controller. The first order transfer function of the process has been adopted as a reference [7]. The derivation to calculate the parameters of ideal internal model control based proportional integral derivative controller is given below.

$$G_p(s) = k_p / (\tau_p s + 1)$$

$G_p(s)$  is the transfer function of the process

$$G_p(s) = 10 / (8s + 1) \quad (56)$$

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The ideal internal model control controller transfer function  $Q(s)$  which includes the filter  $f(s)$  to make a  $Q(s)$  semiproper is given below

$$Q(s) = G_p^{-1}(s) f(s) \quad (57)$$

$$f(s) = 1 / (\lambda s + 1) \quad (58)$$

$$Q(s) = ((8s + 1) / 10) * (1 / (\lambda s + 1)) \quad (59)$$

$\lambda$  is the tuning parameter of the filter  $f(s)$

Take the value of  $\lambda$  as 0.533, which is practically one third of one fifth of time constant and put it in equation (59) to get the value of ideal internal model control controller  $Q(s)$ . The equation becomes

$$Q(s) = ((8s + 1) / 10) * (1 / 0.53s + 1)$$

$$Q(s) = (8s + 1) / (5.33s + 10) \quad (60)$$

From the above equations, the value for the proportional integral tuning parameters is given by

$$k_c = \tau_p / k_p \lambda$$

$$k_c = 8 / 5.3 = 1.5$$

$$\tau_i = \tau_p = 8$$

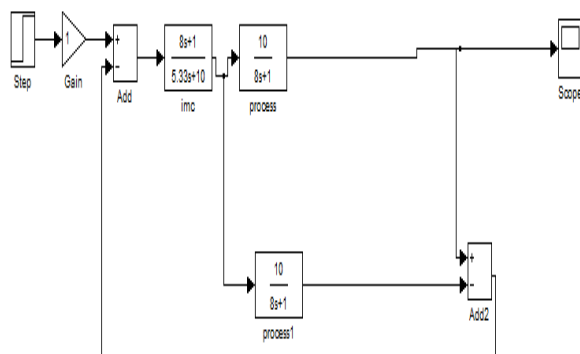
So the transfer function of proportional integral controller is now given by

$$G_c(s) = k_c ((\tau_i s + 1) / (\tau_i s))$$

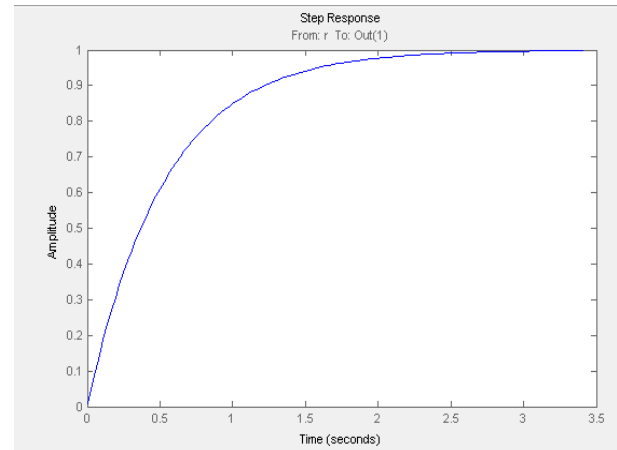
$$G_c(s) = 1.5 ((8s + 1) / (8s))$$

The above transfer function without delay and disturbance results in proportional integral control only.

The Simulink block diagram of ideal IMC based proportional integral derivative controller is shown in figure 4



**Figure 4:** Block diagram of ideal internal model control based proportional integral derivative design



**Figure 4.1:** Unit step response of ideal internal model control based proportional integral derivative controller with no disturbance and time delay

### c. Simulation of Internal Model Control Based Proportional Integral Derivative Design for a First order with Time Delay + First order disturbance:

The transfer function of an internal model control based proportional integral derivative controller for a first order with time delay plus first order disturbance is given below. The transfer function is taken from the reference papers [9]. A first order Pade's approximation is used for time delay.

$$G_p(s) = ((100 / (100s + 1)) * e^{-\theta s}) \quad (61)$$

$$G_d(s) = (1 / (30s + 1)) \quad (62)$$

$G_d(s)$  is the disturbance

$\theta$  = time delay

So,  $\theta = 1$

As there is time delay so first order Pade's approximation is used which is given by

$$e^{-\theta s} = (-0.5s + 1) / (0.5s + 1) \quad (63)$$

$$G_p(s) = (100 / (100s + 1)) * ((-0.5s + 1) / (0.5s + 1)) \quad (64)$$

Factor out the noninvertible elements to avoid the bad part all-pass

$$p^-(s) = 10 / (100s + 1) (0.5s + 1) \quad (65)$$

$$p^+(s) = (-0.5s + 1) \quad (66)$$

Now the value of  $f(s) = 1 / (\lambda s + 1)^2$  to make the controller semiproper

$$Q(s) = ((100s + 1) (0.5s + 1) / 100) * 1 / (\lambda s + 1)^2 \quad (67)$$

Take the value of  $\lambda$  as 20, which is having range  $\lambda > 0.2\tau_p$ . But practically the initial values of  $\lambda$  lie between one third to one fifth of time constant. Put the value of  $\lambda$  in equation (67) to get the value of IMC controller  $Q(s)$ . The equation becomes

$$Q(s) = (50s^2 + 100.5s + 1) / (400s^2 + 40s + 1) \quad (68)$$

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From the above equation (68), the value for the proportional integral derivative tuning parameters is given by

$$k_c = (\tau_p + 0.5\theta) / k_p (\lambda + 0.5\theta)$$

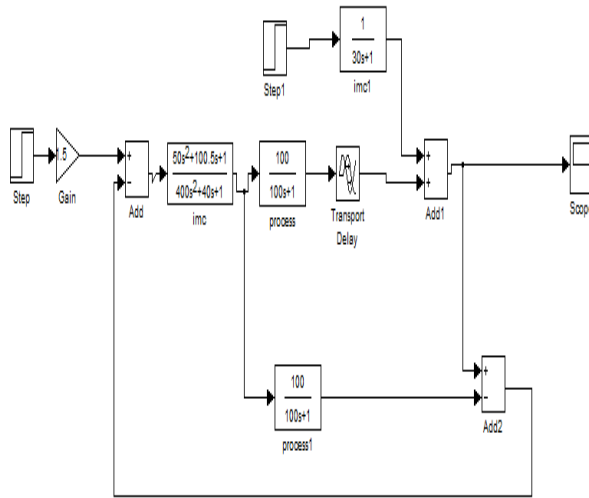
$$k_c = (100 + .5) / 100 (20 + 0.5) = 0.049 \quad (69)$$

$$\tau_i = \tau_p + 0.5\theta = 100.5 \quad (70)$$

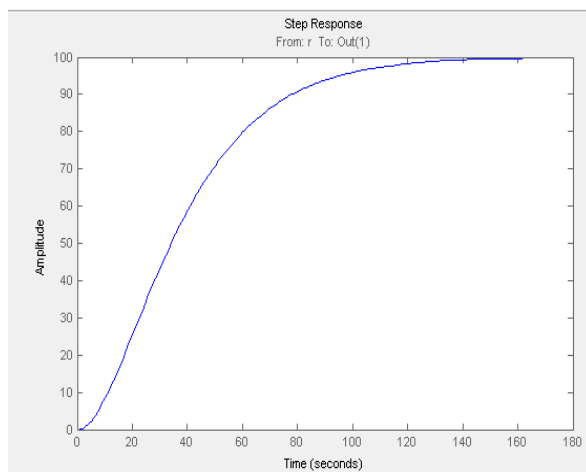
$$\tau_d = \tau_p \theta / 2 \tau_p + \theta = 0.5 \quad (71)$$

So the transfer function of proportional integral derivative controller is now given by

$$G_c(s) = Q(s) / (1 - G_p(s) Q(s)) \quad (72)$$



**Figure 4.2:** Block diagram of internal model control based proportional integral derivative controller for a first order with time delay plus first order disturbance



**Figure 4.3:** Unit step response of internal model control based proportional integral derivative controller for a first order with time delay plus first order disturbance

### Simulation of Internal Model Control Based Proportional Integral Derivative Design for a Second order with time delay plus first order disturbance:

The transfer function of an internal model control based proportional integral derivative controller for a second order with time plus first order disturbance is given below. The transfer function is taken from the reference papers [9]. As it is difficult to implement internal model control controller directly to higher order system due to increased complexity so it is reduced to a low order model. The method used to reduce the model is given by the half rule. According to this rule the largest neglected denominator time constant lag is distributed evenly to the effective delay and the smallest retained time constant. In it disturbance of first order is taken which is given below. A first order Pade's approximation is used for time delay.

$$G_p(s) = (2 / (10s + 1) (5s + 1)) * e^{-\theta s} \quad (73)$$

$$G_d(s) = (1 / (30s + 1)) \quad (74)$$

For a first order model  $\tau_2 = 0$  and the above parameters is given as

$$\tau_1 = \tau_10 + (\tau_20 / 2); \quad \theta = \theta_0 + (\tau_20 / 20) + \sum_{i \geq 3} \tau_i0$$

$$\tau_1 = 12.5; \quad k = 2; \quad \tau_2 = 0; \quad \theta = 3.5 \quad (75)$$

By using half rule reduced model is given as

$$G_p(s) = (2 / (12.5s + 1)) * e^{-3.5\theta s} \quad (76)$$

As there is time delay so first order Pade's approximation is used which is given by

$$e^{-3.5\theta s} = (-1.75s + 1) / (1.75s + 1) \quad (77)$$

$$G_p(s) = (2 / (12.5s + 1)) * ((-1.75s + 1) / (1.75s + 1)) \quad (78)$$

Factor out the noninvertible elements to avoid the bad part all-pass

$$p_-(s) = 2 / (12.5s + 1) (1.75s + 1) \quad (79)$$

Now the value of  $f(s) = 1 / (\lambda s + 1)^2$  to make the controller semiproper

$$Q(s) = ((12.75s + 1) (1.75s + 1) / 2) * 1 / (\lambda s + 1)^2 \quad (80)$$

Take the value of  $\lambda$  as 3, which is having range  $\lambda > 0.2\tau_p$ . But practically the initial values of  $\lambda$  lie between one third to one fifth of time constant. Put the value of  $\lambda$  in equation (80) to get the value of



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internal model control controller Q(s). The equation becomes

$$Q(s) = (21.875s^2 + 14.25s + 1) / (18s^2 + 12s + 2) \dots(81)$$

The value for the proportional integral derivative tuning parameters is given by

$$\tau_c = \theta = 3.5 \quad (82)$$

$$k1 = k / \tau10 = 0.2 \quad (83)$$

$$kc = (1 / k1) * (1 / \tau_c + \theta) = 0.312 \quad (84)$$

$$\tau_i = 8\theta = 8 * 3.5 = 28 \quad (85)$$

$$\tau_D = \tau2 = 5 \quad (86)$$

By getting the above values PID transfer function can be evaluated

### Simulation of Internal Model Control Based Proportional Integral Derivative Design for a third order with time Delay plus first order disturbance:

The transfer function of an internal model control based proportional integral derivative controller for a third order with time delay plus first order disturbance is given below. The transfer function is taken from the reference papers [12]. As it is difficult to implement internal model controller directly to higher order system due to increased complexity so it is reduced to a low order model. The method used to reduce the model is given by the half rule. According to this rule the largest neglected denominator time constant lag is distributed evenly to the effective delay and the smallest retained time constant. In it disturbance of first order is taken which is given below. A first order Pade's approximation is used for time delay.

$$Gp(s) = (2 / (2s + 1) (s + 1)^2) * e^{-\theta s} \quad (85)$$

$$Gd(s) = (1 / (30s + 1)) \quad (86)$$

For a first order model  $\tau2 = 0$  and the above parameters is given as

$$\tau1 = \tau10 + (\tau20 / 2); \quad \theta = \theta \square + (\tau20 / 2) + \sum_{i \geq 3} \tau i0$$

$$\tau1 = 2.5 ; k = 2 ; \tau2 = 0 ; \theta = 1.5 \dots\dots(87)$$

By using half rule reduced model is given as

$$Gp(s) = (2 / (2.5s + 1)) * e^{-1.5\theta s} \quad (88)$$

As there is time delay so first order Pade's approximation is used which is given by

$$e^{-3.5\theta s} = (-0.75s + 1) / (0.75s + 1) \quad (89)$$

$$Gp(s) = (2 / (2.5s + 1)) * ((-0.75s + 1) / (0.75s + 1)) \quad (90)$$

Factor out the noninvertible elements to avoid the bad part all-pass

$$p(s) = 2 / (2.5s + 1) (0.75s + 1) \quad (91)$$

Now the value of  $f(s) = 1 / (\lambda s + 1)^2$  to make the controller semiproper

$$Q(s) = ((2.75s + 1) (0.75s + 1) / 2) * 1 / (\lambda s + 1)^2 \quad (92)$$

Take the value of  $\lambda$  as 1, which is having range  $\lambda > 0.2\tau_p$ . But practically the initial values of  $\lambda$  lie between one third to one fifth of time constant. Put the value of  $\lambda$  in equation (4.30) to get the value of IMC controller Q(s). The equation becomes

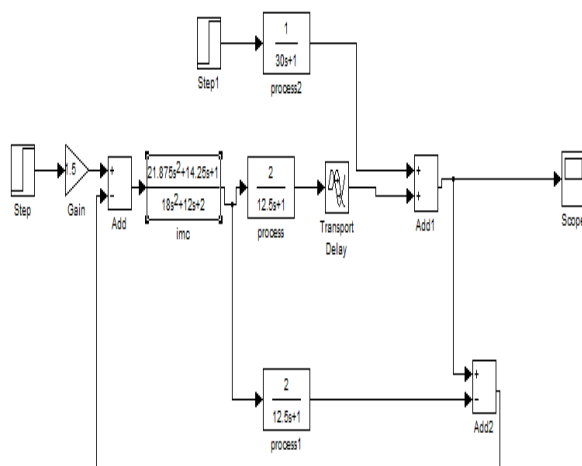


Figure 4.4: Block diagram of internal model control based proportional integral derivative controller for a second order with time delay plus first order disturbance

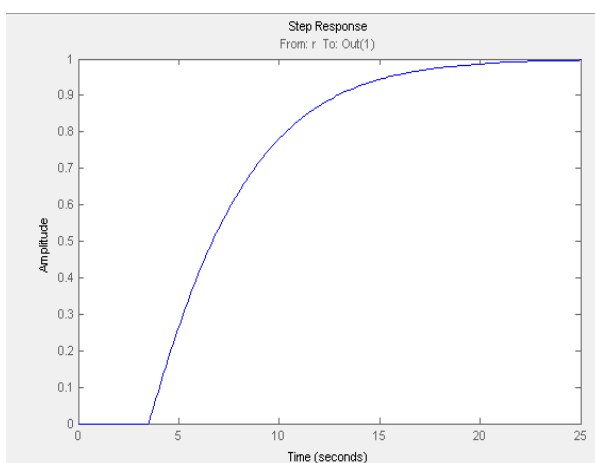


Figure 4.5: Unit step response of internal model control based proportional integral derivative controller for a second order with time delay plus first order disturbance

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$$Q(s) = (1.875s^2 + 3.25s + 1) / (2s^2 + 4s + 2) \quad (93)$$

The value for the proportional integral derivative tuning parameters is given by

$$\tau_c = \theta = 1.5 \quad (94)$$

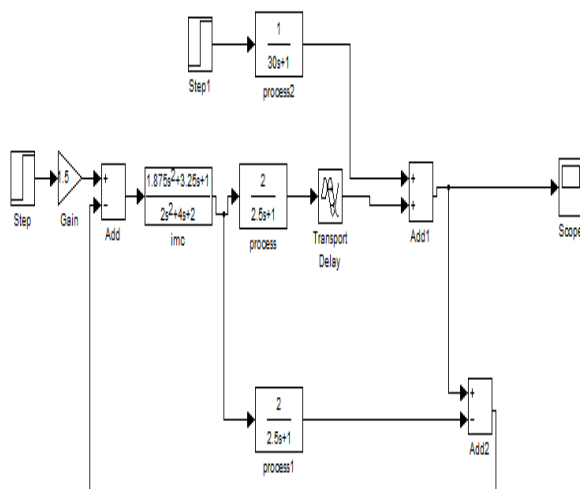
$$k_1 = k / \tau_{10} = 1 \quad (95)$$

$$k_c = (1 / k_1) * (1 / \tau_c + \theta) = 0.33 \quad (96)$$

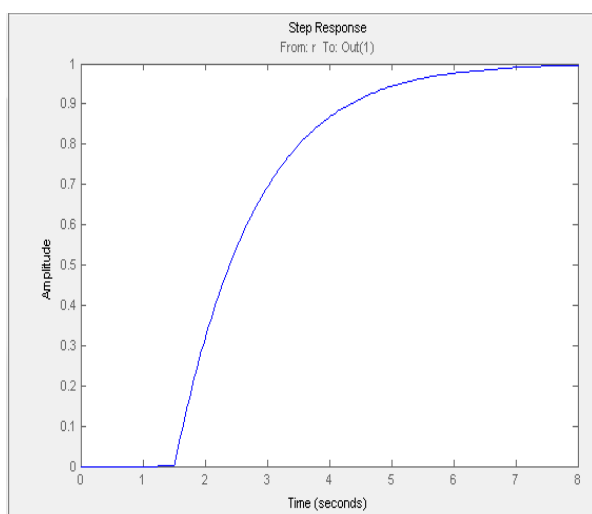
$$\tau_i = 8\theta = 8 * 1.5 = 12 \quad (97)$$

$$\tau_D = \tau_2 = 1 \quad (98)$$

By getting the above values proportional integral derivative transfer function can be evaluated.



**Figure 4.6:** Simulink block diagram of internal model control based proportional integral derivative controller for the third order with time delay plus first order disturbance



**Figure 4.7:** Unit step response of internal model control based proportional integral derivative controller for the third order with time delay plus first order disturbance

Various tuning parameters of internal model control based proportional integral derivative design based on different orders of transfer functions obtained from above are shown in table 2

**Table 2:** Various tuning parameters of internal model control based proportional integral derivative design based on different orders of transfer functions

Orders of Transfer functions	$k_c$	$\tau_i$	$\tau_d$	$\tau_c$
Ideal IMC based PID	1.5	8	-	-
First order IMC based PID	0.049	100.5	0.5	-
Second order IMC based PID	0.312	28	5	3.5
Third IMC based PID	0.33	12	1	1.5

## 4. CONCLUSION

The work discussed here is based on the PID controller design using IMC with time delays which affects the output of a system. So Pade's approximation for the time delays in internal model control based proportional integral derivative controller design is used along with half rule to handle the complex models by approximating the remaining high order dynamics by an effective delay. By comparing the figures of unit step responses it is proved that the internal model control based proportional integral derivative controller will not give the same results as the internal model control strategy because of approximation used for Delay time. And also various tuning parameters of PID have been found based on the different orders of transfer functions. The standard internal model control filter from  $f(s) = 1 / (\lambda s + 1)$  shows good set point tracking. Thus internal model control based proportional integral derivative controller is able to compensate for disturbances and model uncertainty while open loop control is not. Internal model controller is also detuned to assure stability even if there is model uncertainty. The internal model control based proportional integral derivative controller design is conventional controller. So due to speed in their execution, accuracy of control, ease of configuration, low energy consumption, probability etc, artificial intelligence based controllers such as

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Fuzzy logic based controllers and Artificial Neural Network based controller can be used

## REFERENCES

- [1] C. E. Garcia, "M. Morari, Internal Model Control. 1. A Unifying Review and Some New Results", *Industrial & Engineering Chemistry Process Design and Development*, Vol. 21, pp.308-323, 1982.
- [2] K. J. Aström, T. Hagglund, "PID Controllers: Theory, Design, and Tuning", 2nd ed, Instrument Society of America, 1995.
- [3] Ian G. Horn, Jeffery R. Arulandu, Christopher J. Gombas, Jeremy G. VanAntwerp, and Richard D. Braatz, "Improved filter design in internal model control", *Industrial Engineering Chemical Resources*, Vol.no 36, pp.3437-3441, 1996.
- [4] Syder J, T Heeg, and A O'Dwyer, "Dead-time Compensators: Performance and Robustness Issues", *European Journal of Control*, pp. 166-171, 2000.
- [5] Sigurd Skogestad, "Probably the best simple PID tuning rules in the world", *Journal of Process Control*, pp. 1-27, 2001.
- [6] Sigurd Skogestad, "Simple Analytic Rules for Model Reduction and PID Cotroller tuning", *Journal of Process Control*, Vol. no 13, pp. 291-309, 2003.
- [7] B wayne Bequette, "Process Control: Modelling, Design, and Simulation", Prentice Hall, pp. (198-206),(294-297),(262-264) 2003.
- [8] A. O'Dwyer, "Handbook of PI and PID Controller Tuning Rules", Imperial College Press, London, pp. 37, 2003
- [9] M. Shamsuzzoha and Moonyong Lee, "IMC-PID Controller Design for Improved Disturbance Rejection of Time Delayed Processes", *Industrial Engineering Chemical Resources*, Vol. no. 46, pp. 2077-2091, 2007.
- [10] R. Farkh, K. Laabidi and M. Ksouri, "PI Control for Second Order Delay System with Tuning Parameter Optimization", *International Journal of Electrical and Electronics Engineering*, pp. 1-7, 2009.
- [11] Susmita Das, Ayan Chakraborty, Jayanta Kumar Ray, Soumyendu Bhattacharjee, Dr. Biswarup Neogi, "Study on Different Tuning Approach with Incorporation of Simulation Aspect for Z-N (Ziegler-Nichols) Rules", *International Journal of Scientific and Research Publications*, Vol. no 2, Issue 8, pp. 1-5, 2012.
- [12] Linkan Priyadarshini, J.S Lather, "Design of IMC-PID Controller for a Higher order system and its Comparison with Conventional PID Controller", *International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering*, Vol. 1, Issue 3, pp.108-112, 2013.
- [13] QIN Gang, SONG Le, HU Ling, "A Practical Application of IMC-PID Controller in Unmanned Vehicle", *Telkonnika*, Vol. 11, No. 6, pp. 3228-3235, e-ISSN: 2087-278X, 2013.