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Implementation and Analysis of Channel Estimation in OFDM System

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Abstract: OFDM is becoming widely applied in wireless communication systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay. Channel estimation in OFDM is usually performed with the aid of pilot symbols. For such systems pilot-symbol assisted modulation (PSAM) on flat fading channels involves the sparse insertion of known pilot symbols in a stream of data symbols. The concept of PSAM in OFDM systems also allows the use of the frequency correlation properties of the channel. There are two main problems in designing estimators for Wireless OFDM system. The first problem is the arrangement of pilot information, where pilot means the reference signal used by both transmitters and receivers. The second problem is the design of an estimator with both low complexity and good channel tracking ability. The present work addresses channel estimation based on the Minimum Mean Square Error (MMSE) and Least Square (LS) criteria and also considers time-domain channel statistics. It presents an optimal criterion for the pilots, and corresponding optimal designs enabling complexity reductions. Using a general model for a slowly fading channel, the MMSE and LS estimators and a method for modifications compromising between complexity and performance is presented. The symbol error rate for system is estimated by means of simulation results.

Keywords: OFDM, LS Estimator, MMSE estimator, Channel estimation.

1. INTRODUCTION

Currently, orthogonal frequency-division multiplexing (OFDM) systems [1] are subject to significant investigation. Since this technique has been adopted in the European digital audio broadcasting (DAB) system [2], OFDM signaling in fading channel environments has gained a broad interest. For instance, its applicability to digital TV broadcasting is currently being investigated [3]. The use of differential phase-shift keying (DPSK) in OFDM systems avoids the tracking of a time varying channel. However, this will limit the number of bits per symbol and results in a 3 dB loss in *signal-to-noise ratio* (SNR) [4]. If the receiver contains a channel estimator, multi-amplitude signalling schemes can be used. In [5] and [6], 16-QAM modulation in an OFDM system has been investigated. In the design of wireless OFDM systems, the channel is usually assumed to have a finite-length

impulse response. A cyclic extension, longer than this impulse response, is put between consecutive blocks in order to avoid interblock interference and preserve orthogonality of the tones. Generally, the OFDM system is designed so that the cyclic extension is a small percentage of the total symbol length. This paper discusses channel estimation techniques in wireless OFDM systems that use this property of the channel impulse response. We describe the system model and discuss the minimum mean-square error (MMSE) and least-squares (LS) channel estimators. The MMSE estimator has good performance but high complexity. The LS estimator has low complexity, but its performance is not as good as that of the MMSE estimator. We present modifications to both MMSE and LS estimators that use the assumption of a finite length impulse response. The performance is presented both in terms of mean-square error (MSE) and symbol error rate (SER).[7]

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2. LITERATURE REVIEW

Orthogonal Frequency Division Multiplexing (OFDM) based systems are strong candidates for an air interface of future 4G mobile wireless systems, which provide high data rates and high mobility. The use of multi-amplitude signaling schemes in wireless OFDM system requires the tracking of fading radio channel. OFDM first proposed in the mid 1960s is a special case of multi-carrier modulation (MCM), which uses the principle of transmitting data by dividing the stream into several parallel bit streams and modulating each of these data streams onto individual, so-called sub carriers. The use of Differential Phase-Shift Keying (DPSK) in OFDM systems avoids the tracking of a time varying channel. However, this will limit the number of bits per symbol and results in a 3 dB loss in Signal-to-Noise Ratio (SNR).[8]

Most documented channel estimation concepts consist of two steps, one or both of which use the correlation of the channel. First, the attenuations at the pilot positions are measured and possibly smoothed using the channel correlation. These measurements then serve to estimate (interpolate) the complex-valued attenuations of the data symbols in the second step. This second step uses the channel correlation properties either with interpolation filters or with a decision-directed scheme. Depending on the pilot arrangement the estimation strategies diverge in this second step. Hoher, for instance, proposes a scattered pilot pattern. The interpolation in the presented scheme uses channel measurements with two FIR Wiener filters. The first Wiener filter interpolates and smoothes the channel attenuations in frequency. A second Wiener filter then interpolates and smoothes the channel attenuations in time. This scheme exploits the channel correlation properties in the design of the interpolating Wiener filters. In general the correlation properties and the SNR needed to design the estimator are not known. Therefore, Hoher proposes to design the estimator for fixed, assumed values of the channel correlations and SNR.

System Description

We will consider the system shown in Fig. 1, where x_k are the transmitted symbols, $g(t)$ is the channel impulse response, $e_n(t)$ is the white complex Gaussian channel noise and y_k are the received symbols. The transmitted symbols x_k are taken from a multi-amplitude signal constellation. The D/A and A/D converters contain ideal low-pass filters with bandwidth $1/T_s$, where T_s is

the sampling interval. A cyclic extension of time length T_G (not shown in Fig. 1 for reasons of simplicity) is used to eliminate inter-block interference and preserve the orthogonality of the tones. We treat the channel impulse response $g(t)$ as a time limited pulse train of the form

$$g(t) = \sum_m \alpha_m \delta(t - \tau_m T_s) \quad (1)$$

where the amplitude α_m are complex valued and $0 \leq \alpha_m T_s \leq T_G$, i.e. the entire impulse response lies inside the guard space.

The system is then modeled using the N -point discrete-time Fourier transform (DFT _{N}) as

$$y = \text{DFT}_N(\text{IDFT}_N(x) \otimes \frac{g}{\sqrt{N}} + n') \quad (2)$$

where \otimes denotes cyclic convolution, $x = [x_0 \ x_1 \ \dots \ x_{N-1}]^T$, $y = [y_0 \ y_1 \ \dots \ y_{N-1}]^T$, $n' = [n'_0 \ n'_1 \ \dots \ n'_{N-1}]^T$ is a vector of complex gaussian variables. $g = [g_0 \ g_1 \ \dots \ g_{N-1}]^T$ is determined by the cyclic equivalent of

sinc functions. The vector $\frac{g}{\sqrt{N}}$ is the observed impulse channel response after sampling the frequency response of $g(t)$.

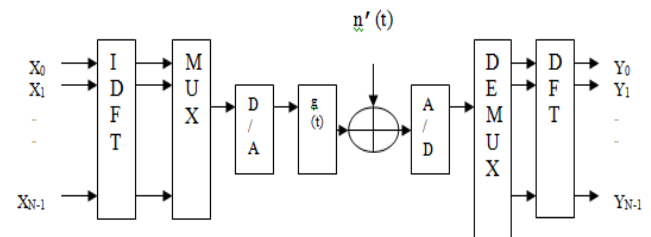


Figure1: Baseband OFDM System

The system described by (2) can be written as a set of N independent Gaussian channels

$$y_k = h_k x_k + n_k; \quad k = 0 : : N-1; \quad (3)$$

where h_k is the complex channel attenuation given by $\mathbf{h} = [h_0 \ h_1 \ \dots \ h_{N-1}]^T = \text{DFT}_N(\mathbf{g})$ and $\mathbf{n} = [n_0 \ n_1 \ \dots \ n_{N-1}]^T = \text{DFT}_N(\mathbf{n}')$ is an i.i.d. complex zero-mean Gaussian noise vector. As a matter of convenience, we write (3) in matrix notation

$$y = \mathbf{X}\mathbf{F}\mathbf{g} + \mathbf{n}; \quad (4)$$

where \mathbf{X} is a matrix with the elements of x on its diagonal and \mathbf{F} is the DFT-matrix

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MMSE and LS Estimators

If the channel vector \mathbf{g} is Gaussian and uncorrelated with the channel noise \mathbf{n} , the MMSE estimate of \mathbf{g} becomes [9]

$$\mathbf{g}'_{\text{MMSE}} = \mathbf{R}_{\mathbf{g}\mathbf{y}} \mathbf{R}_{\mathbf{y}\mathbf{y}}^{-1} \mathbf{y} \quad (5)$$

where

$$\mathbf{R}_{\mathbf{g}\mathbf{y}} = \mathbf{E} \{ \mathbf{g}\mathbf{y}^H \} = \mathbf{R}_{\mathbf{g}\mathbf{g}} \mathbf{F}^H \mathbf{X}^H \quad (6)$$

$$\mathbf{R}_{\mathbf{y}\mathbf{y}} = \mathbf{E} \{ \mathbf{y}\mathbf{y}^H \} = \mathbf{X}\mathbf{F}\mathbf{R}_{\mathbf{g}\mathbf{g}}\mathbf{F}^H\mathbf{X}^H + \sigma_n^2 \mathbf{I}_N \quad (7)$$

are the cross-covariance matrix between \mathbf{g} and \mathbf{y} and the auto covariance matrix of \mathbf{y} . Further $\mathbf{R}_{\mathbf{g}\mathbf{g}}$ is the auto covariance matrix of \mathbf{g} and σ_n^2 denotes the noise variance. These two quantities are assumed to be known. Since the columns in \mathbf{F} are orthonormal, $\mathbf{g}'_{\text{MMSE}}$ generates the frequency domain MMSE estimate $\mathbf{h}'_{\text{MMSE}}$ by

$$\mathbf{h}'_{\text{MMSE}} = \mathbf{F} \mathbf{g}'_{\text{MMSE}} = \mathbf{F} \mathbf{Q}'_{\text{MMSE}} \mathbf{F}^H \mathbf{X}^H \mathbf{y} \quad (8)$$

where, $\mathbf{Q}'_{\text{MMSE}}$ can be shown to be

$$\mathbf{Q}'_{\text{MMSE}} = \mathbf{R}_{\mathbf{g}\mathbf{g}} [(\mathbf{F}^H \mathbf{X}^H \mathbf{X} \mathbf{F})^{-1} \sigma_n^2 + \mathbf{R}_{\mathbf{g}\mathbf{g}}]^{-1} (\mathbf{F}^H \mathbf{X}^H \mathbf{X} \mathbf{F})^{-1} \quad (9)$$

If \mathbf{g} is not Gaussian, $\mathbf{h}'_{\text{MMSE}}$ is not necessarily a minimum mean-square error estimator.[10] It is however the best linear estimator in the mean-square error sense. In either case (\mathbf{g} , Gaussian or not) the channel estimate is to be denoted as $\mathbf{h}'_{\text{MMSE}}$. The LS estimator for the cyclic impulse response \mathbf{g} minimizes $(\mathbf{y} - \mathbf{X}\mathbf{F}\mathbf{g})^H (\mathbf{y} - \mathbf{X}\mathbf{F}\mathbf{g})$ and generates

$$\mathbf{h}'_{\text{LS}} = \mathbf{F}\mathbf{Q}'_{\text{LS}} \mathbf{F}^H \mathbf{X}^H \mathbf{y}, \quad (10)$$

where,

$$\mathbf{Q}'_{\text{LS}} = (\mathbf{F}^H \mathbf{X}^H \mathbf{X} \mathbf{F})^{-1} \quad (11)$$

Above equation reduces to

$$\mathbf{h}'_{\text{LS}} = \mathbf{X}^{-1} \mathbf{y}, \quad (12)$$

The LS estimator is equivalent to what is also referred to as the zero forcing estimators. Both estimators LS and MMSE have their drawbacks. The MMSE estimator suffers from a high complexity, whereas the LS estimate has a high mean-square error.

3. NEW PROPOSED SCHEME

Modified MMSE and LS Estimators:

The MMSE estimator requires the calculation of an $N * N$ matrix $\mathbf{Q}'_{\text{MMSE}}$, which implies a high complexity when N is large. A straightforward way of decreasing

the complexity is to reduce the size of $\mathbf{Q}'_{\text{MMSE}}$. Most of the energy in \mathbf{g} is contained in, or near, the first $L = T_G / T_S$ taps. Therefore a modification of the MMSE estimator is studied, where only the taps with significant energy are considered. The elements in $\mathbf{R}_{\mathbf{g}\mathbf{g}}$ corresponding to low energy taps in \mathbf{g} are approximated by zero. If the first L taps of \mathbf{g} are to be taken, and set $\mathbf{R}_{\mathbf{g}\mathbf{g}}(r,s) = 0$ for $r,s \notin [0,L-1]$, then $\mathbf{Q}'_{\text{MMSE}}$ is effectively reduced to an $L * L$ matrix. If the matrix \mathbf{T} denotes the first L columns of the DFT-matrix \mathbf{F} and $\mathbf{R}'_{\mathbf{g}\mathbf{g}}$ denotes the upper left $L * L$ corner of $\mathbf{R}_{\mathbf{g}\mathbf{g}}$, the modified MMSE estimator becomes

$$\mathbf{h}'_{\text{MMSE}} = \mathbf{T} \mathbf{Q}'_{\text{MMSE}} \mathbf{T}^H \mathbf{X}^H \mathbf{y} \quad (13)$$

Where,

$$\mathbf{Q}'_{\text{MMSE}} = \mathbf{R}'_{\mathbf{g}\mathbf{g}} [(\mathbf{T}^H \mathbf{X}^H \mathbf{X} \mathbf{T})^{-1} \sigma_n^2 + \mathbf{R}'_{\mathbf{g}\mathbf{g}}]^{-1} [(\mathbf{T}^H \mathbf{X}^H \mathbf{X} \mathbf{T})^{-1}] \quad (14)$$

As an OFDM system is usually designed so that L is a small fraction of N . Thus, the complexity of the MMSE estimator will decrease considerably. Although the complexity of the LS estimator does not prompt for modifications, its performance in terms of mean-square error can be improved for a range of SNRs by following general concept as above. [11]

The LS estimator does not use the statistics of the channel. Intuitively, excluding low energy taps of \mathbf{g} will to some extent compensate for this shortcoming since the energy of \mathbf{g} decreases rapidly outside the first L taps, whilst the noise energy is assumed to be constant over the entire range.

Taking only the first L taps of \mathbf{g} into account, thus implicitly using channel statistics, the modified LS estimator becomes

$$\mathbf{h}'_{\text{LS}} = \mathbf{T} \mathbf{Q}'_{\text{LS}} \mathbf{T}^H \mathbf{X}^H \mathbf{y} \quad (15)$$

$$\text{where, } \mathbf{Q}'_{\text{LS}} = (\mathbf{T}^H \mathbf{X}^H \mathbf{X} \mathbf{T})^{-1} \quad (16)$$

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4. CONCLUSION AND FUTURE SCOPE

Simulation

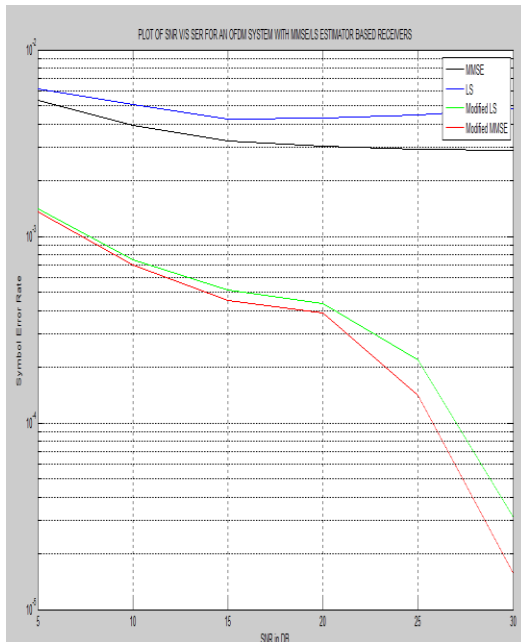


Figure 2: SNR v/s SER for different estimators

- Study of channel estimation based on block type pilot arrangement is presented, and it is observed that this type of arrangement performs better when the channel is changing slowly
- The MMSE estimator has good performance but high complexity. The LS estimator has low complexity, but its performance is not as good as that of the MMSE estimator.
- The MMSE estimator suffers from a high complexity, whereas the LS estimate has a high mean-square error

Suggestions for Future work:

- For future research the method to determine the channel taps and improved efficiency of the system may be considered
- Also the channel can be estimated by using different modulation schemes and their performance can be compared
- We can also go for different methods suitable for estimating channel during fast fading by

determining the number of pilots and their positions

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