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De-noising of Signal Though Wavelets

Pooja Rani¹, Vikas Sandhu², Viney Dhawan³

¹KITM, Kunjpura, Karnal
poojakandhol@gmail.com

² KITM, Kunjpura, Karnal
vikassandhu20@gmail.com

³ KITM, Kunjpura, Karnal
vineydhawan2008@gmail.com

Abstract: Noise has been a primary deterrent in signal transmission and processing. It results in faulty information after processing the signals reducing their usability. In our work, different kind of wavelets (such as haar, daubechies, coif, etc) was used to filter out the noise from different kind of signals (such as electrical signals, sine waves, EGC etc) for this purpose we used the wavelets tool box of MATLAB software, and try to find out the best wavelets for these signals.

Keywords: Wavelets, de-noising, EGC, daubechies, CWT, DWT.

1. INTRODUCTION

A signal is defined as any physical quantity that varies with time, space and any other parameters (independent variables). Digital signal processing is an area of science and engineering that has developed rapidly over the past 40 years. DSP gives the proper solution for the mostly signal processing problems. [2] Signals always have some noise associated with them, rarely do we find signals in "real life" situations that are free from noise and can be directly employed for extracting information. Noise can result in an output which may not be intended or not the characteristic of the quality being observed, giving rise to faults in the system of which the signals is a component. It can also cause judgmental errors if the signals is being directly observed and the impact can range from being minute in some cases to destructive in certain critical systems like ECG machines [9].

Hence, it's impotent that the noise should be removed from the signals. Most of the times the noise found in the signals is of higher frequency as compared to the signals produced by the quantity being measured or represented. It's, therefore, of almost impotence that the noise from the signals is removed to the optimal extents. The problem of noise in the signals is not new. Various solutions have been proposed and are currently being employed a numbers of systems. The earlier method of signals analysis was based on time domains method. But this is not always sufficient to study all features of ECG signals, so, the frequency representations of a signals is required to accomplish this, FFT (fast Fourier transform) technique is applied. But the unavoidable limitation of this FFT is that the technique failed to provide the information's regarding the exact locations of frequency components in time. As the frequency content of the signals varies with time, the need for an accurate description of the signals frequency contents according to their locations in the time is essential. This justify the used of time frequency representation of the signals. [10]

The immediate tool available for this purpose is the short term Fourier transform (STFT) but the major drawback of this STFT is that it's time. A frequency precision is not optima's.

Hence we used a more suitable technique to overcome this drawback; among the various time frequency transformations the wavelets transformation is found to be a simple and more valuable. [6] In recent years, wavelets transform has developed rapidly. It breaks through the limitation of Fourier transform in time domains. It can analyze the components of the signals appointed by the user at frequency band and time segment. It has fine localized character in time and frequency domain. Using gradually fine sampling step of time or frequency domains, wavelets can focus on any details of the signals. Now wavelet transform is widely used in signals processing. In our approach, wavelets transform and inverse wavelets transform were used. Since there are a huge numbers of wavelets families having several different wavelets having high number of vanishing moments and capable of representing complex polynomials, it was not different to find wavelet which was similar to the signals being produced. When transform with the similar wavelet, the disturbance caused in the original signals were minimize which reduced the overhead. The key point in this approach were analyzing the signals to find a suitable wavelet, applying the transform, performing threshold operations and inverse transforming it. [5]

2. MATERIALS AND METHODS

2.1 Wavelets:

Wavelets are mathematical functions that satisfy certain mathematical requirements and are used in representing data or other functions. In other words wavelets are the mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. [1] The fundamental idea behind wavelet is to analyze according to scale. The wavelet analysis procedure is to adopt a wavelet prototype function, called an analyzing wavelet or mother wavelet. In wavelet analysis, the scale that we use to look at data plays a special role. Wavelet algorithms process data at different scales or resolutions. [13]

INTERNATIONAL JOURNAL FOR ADVANCE RESEARCH IN ENGINEERING AND TECHNOLOGY

WINGS TO YOUR THOUGHTS.....

2.2 Wavelet transforms:

Wavelet transforms are classified as continuous wavelets transform (CWT) and discrete wavelet transform (DWT). The finite oscillatory nature of the wavelet makes them extremely useful in real life situations in which signals are not stationary. Wavelet transform” variable time frequency” resolution which is the hallmarks of wavelet transforms.[3]

- A wavelets transform decomposed a signal into basic functions which are known as wavelets.
- Wavelet transform is calculated separately for different segments of the time domain signals at different frequencies resulting in multi-resolution analysis or MRA.
- It is designed in such a way that a product a time resolution and the frequency resolution is constant.
- It's give good time resolution and poor frequency resolution at high frequencies.
- This feature of MRA makes it excellent for signal having high frequency components for short duration and low frequency components for long duration. E.g. noise in signals images, video frames etc.

(a) Discrete wavelet transform

A wavelet transform in which the wavelets are discretely samples are known as discrete wavelet transform.

The DWT gives a multi resolution description of a signal which a very useful in analyzing” real world” signals discrete wavelet transform (DWT) is defined as:

$$DWT(m, n) = 2^{-\frac{m}{2}} \sum_k s(k) \varphi(2^{-m} k - n)$$

The DWT gives a multi-resolution description of a signal which is very useful in analyzing "real-world" signals. Essentially, a discrete multi-resolution description of a continuous-time signal is obtained by a DWT.[7] It converts a series $a_0, a_1, a_2, \dots, a_m$ into one low pass coefficient series known as “approximation” and one high pass coefficient series known as “detail”. Length of each series is $m/2$. In real life situations, such transformation is applied recursively on the low-pass series until the desired number of iterations is reached. Some examples of discrete wavelets are the Haar wavelets, Daubechies wavelets, symmlets etc. For any input comprising of $2n$ numbers, the Haar wavelet transform simply pairs up input values, storing the difference and passing the sum. This process is recursive, pairing up the sums to provide the next scale: finally resulting in $2n - 1$ differences and one final sum and this is done in $O(n)$ time i.e. linear time. The function is not continuous and hence not differentiable. Daubechies wavelets are families of wavelets whose inverse wavelet transforms are adjoint of the wavelet transform i.e. they are orthogonal. They have maximal number of vanishing moments and hence they can represent higher degree polynomial functions. With each wavelet type of this class, there is a scaling function known as “father wavelet” that generates an orthogonal multi-resolution analysis. Daubechies orthogonal wavelets D2-D20 (even index numbers only) are commonly used. The numbers associated with the name refers

to the number ‘N’ of coefficients. Each wavelet has vanishing moments equal to half the number of coefficients. For example, D2 which is the Haar wavelet has one vanishing moment, D4 has two, etc. The number of vanishing moments is what decides the wavelet's ability to represent a signal. For example, D2, with one moment, easily encodes polynomials of one coefficient, or constant signal components. D4 encodes polynomials with two coefficients, i.e. constant and linear signal components etc. The wavelet transform using Daubechies wavelets result in progressively finer discrete samplings using recurrence relations. Every resolution scale is double that of the previous scale. Daubechies derived a family of wavelets, the first of which is the Haar wavelet. Since then interest in this field has shot up and many variations of Daubechies original wavelets have been developed. The discrete wavelet transform has applications ranging from data compression to signal coding. In our research work, Daubechies wavelet was used to filter a noisy signal to extract information from the signal.[8][12].

(b) Continuous wavelets transform

The continuous wavelet transform (CWT) is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function or alternatively as shown in the following equation:

$$CWT(a, \tau) = \frac{1}{\sqrt{a}} \int s(t) \varphi\left(\frac{t-\tau}{a}\right) dt$$

In this equation the parameter “a” is the scaling factor that stretches or compresses the function. The parameter τ is the translation factor that shifts the mother wavelet along the axis. The parameter $s(t)$ is an integrable signal whose sum is to be multiplied by the translated mother wavelet. Mother wavelet is denoted by $\Psi(t)$, which is a function of the scaling and translation factors just as the result of the continuous wavelet is the wider is the basis function transformation CWT.[11]

2.3 Wavelet analysis:

The wavelet analysis of signals is performed using MATLAB software. MATLAB is a high performance, interactive system which allows storing many technical computing problems. The MATLAB software package is provided with wavelet tool box. It's an also a collection of function built on MATLAB technique computing environment.[14] Its provide tools for the analysis and synthesis of signals and image using wavelets packet within the MATLAB domains.[4]

The de-noising procedure can be summarized as follows:

Step 1 - Analysis Step:

Selecting an appropriate wavelet was a very important task in this step. The wavelet chosen should be similar to the signal that has to be filtered to give the best possible results. This “similarity” can be decided on the basis of the cross-correlation between the two functions. We had preferred the Daubechies family of wavelets because of their high number of vanishing moments making them capable of representing

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WINGS TO YOUR THOUGHTS.....

complex high degree Polynomials. The result of our simulations showed that db wavelet provided sufficiently good signal output.

Step 2- Threshold Step:

After applying the selected wavelet transform to the input vector we obtained a numerically transformed vector which had the detail coefficients that are carried from one level to the next as it is and the final left approximation values. To de-noise the signal, the detail coefficients were made '0' after applying the transform. Then, the size of the input vector, which had sampled values of the signal, was known and it was also known that each time the size of the resultant vector had been reduced to half the original size.

Step 3- Inverse Wavelet Transform:

After applying the wavelet transform and threshold procedures, the inverse wavelet transform was applied. The output of this step, as seen in the figure, was the original signal with very less noise as compared with the earlier signal.

2.4 Results and discussion:

In this work we point out the advantage of using wavelets associated with a noise thresholding strategy .Wavelets with their "variable time frequency resolution" and properties such as MRA and high number of vanishing moments provide an effective way to analysis a signal .The process of signal de-noising can be performed in quick time following this approach. The simulation that had been performed reveal that wavelets can be used to separate different frequency components of the signal efficiently. After separating the signal into components, the unwanted signal components can be removed by setting the detail coefficients.The inverse transform can be applied on the semi processed vector to get back the original signal which is free from noise components.

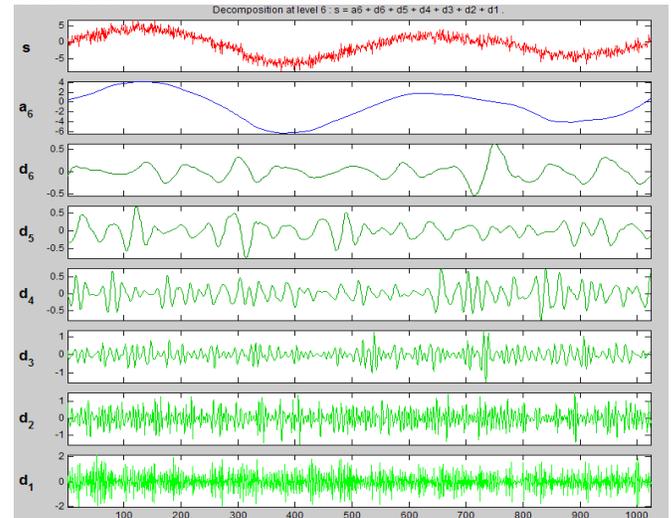


Figure 2: Sine wave with noise, after applying the wavelet transform to the signal: a6 is the approximation at 6th level, d1-d6 are the detail coefficients at respective levels which are set to '0' in the threshold step.

Table 1: Wavelet specification for sine wave

Wavelet	SNR (db)	Compression ratio	Retained energy (%)	Zeros (%)
Haar	28.4	0.63	89.82	89.72
Coif3	31.2	0.89	91.64	91.67
Db4	32.2	0.99	90.82	90.81
Db6	34.6	1.34	90.43	90.48

Figure 3 shows the de-noising of electrical signal. The procedure of de-nosing is same as the sine wave signal. In this work db6 gives the best result for sine wave, and db4 gives the best result for electrical signal .In this work we observed that Daubechies wavelet is the best wavelet for all kinds of signals (which is included in this work) for the de-noising purposed.

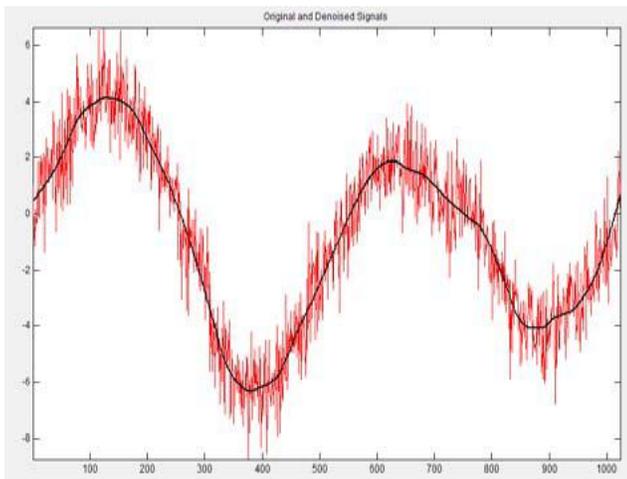


Figure 1: Comparison between the original and filtered signal of sine wave.

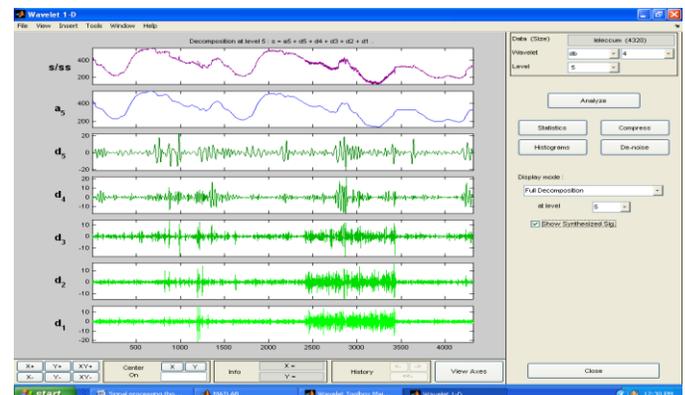


Figure 3: Electrical signal with noise after applying the wavelet transform to the signal: a5 is the approximation at 5th level,d1-d5 are the detail coefficient at respective level which are set to zero in the threshold step.

INTERNATIONAL JOURNAL FOR ADVANCE RESEARCH IN ENGINEERING AND TECHNOLOGY

WINGS TO YOUR THOUGHTS.....

Table 2: Wavelet specification for electrical signal

Wavelet	SNR (db)	Compression Ratio	Retained energy (%)	Zeros (%)
Haar	46.4	0.58	99.92	96.68
Coif	41.3	0.66	99.97	96.84
Sym	40.7	0.71	99.97	96.76
Dd2	38.9	0.86	99.97	96.84
Db4	36.3	1.89	99.97	96.68

We observed that the daubechies wavelet are best are best for all types of signals. For the de-noising purpose figure (1), figure (2) and figure (3) are shown the results. In future we will try to find out the best wavelet family which is suitable for all kinds of signals and images de-noising.

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